

CASIMIR FORCE UNDER THE INFLUENCE OF REAL CONDITIONS

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Abstract

The Casimir force is calculated analytically for configurations of two parallel plates and a spherical lens (sphere) above a plate with account of nonzero temperature, finite conductivity of the boundary metal and surface roughness. The permittivity of the metal is described by the plasma model. It is proved that in case of the plasma model the scattering formalism of quantum field theory in Matsubara formulation underlying Lifshitz formula is well defined and no modifications are needed concerning the zero-frequency contribution. The temperature correction to the Casimir force is found completely with respect to temperature and perturbatively (up to the second order in the relative penetration depth of electromagnetic zero-point oscillations into the metal) with respect to finite conductivity. The asymptotics of low and high temperatures are presented and contributions of longitudinal and perpendicular modes are determined separately. Serving as an example, aluminium test bodies are considered showing good agreement between the obtained analytical results and previously performed numerical computations. The roughness correction is formally included and formulas are given permitting to calculate the Casimir force under the influence of all relevant factors.

1 Introduction

Lately considerable progress had been made both in theoretical and experimental investigation of the Casimir effect. This effect predicted by H.B.G. Casimir¹ more than fifty years ago, consists in the interaction of two neutral, conducting bodies placed in vacuum close to each other. The Casimir effect results from the disturbance by the conducting boundaries of the zero-point electromagnetic oscillations. It plays an important role in various fields of physics such as elementary particle theory, condensed matter physics, atomic physics, gravitation and cosmology, and stimulated new investigations in mathematical physics (see the monographs^{2–5}). Recently the Casimir effect found applications^{6–11} for obtaining rather strong constraints on hypothetical long-range interactions inspired by the physics of extra dimensions, by unified gauge theories, supersymmetry and supergravity. Furthermore, topical nanoelectromechanical devices were proposed^{12–14} which are based on the use of the Casimir force.

In precision experiments on the measurement of the Casimir force^{14–19} different influential factors must be accounted for, such as nonzero temperature, finite conductivity of the boundary metal and surface roughness. Theoretically, each factor was investigated in a number of papers (see, e.g., Refs. 20–24 for the influence of nonzero temperature, Refs. 22, 25–29 for the role of finite conductivity and Refs. 30–36 for the surface roughness). The combined effect of different corrections was discussed in Refs. 23, 24, 35 (for a detailed discussion of this subject see the recent review³⁷).

Investigation of the combined effect of nonzero temperature along with the finite conductivity of the boundary metal proved to be the most complicated task leading to controversial results. The starting point to theoretically describe this effect is the Lifshitz theory³⁸ originally being developed for dielectrics. In order to describe on the base of Lifshitz' theory the Casimir force between plates made of an ideal metal a special prescription was suggested in

Ref. 22 which demands to consider first the limit of infinite dielectric permittivity before setting the frequency equal to zero. Using this prescription, the results of the Lifshitz theory for ideal metals agreed with the results obtained by the application of quantum field theory using the idealized boundary conditions.^{20,21}

In the last year, several authors attempted to apply the Lifshitz theory to calculate the Casimir force between plates made of real metals. In Ref. 23 the plasma model was used to describe the dependence of dielectric permittivity on frequency. In Ref. 39 the Drude dielectric function (being a generalization of the plasma model by taking into account the relaxation processes) was substituted into the Lifshitz formula. However, in the limit of zero relaxation frequency the results of Ref. 39 do not coincide with those of Ref. 23 although Drude's model turns into plasma model in this limit. In Ref. 24 the results of Ref. 23 obtained using the plasma model were independently confirmed and doubts were casted on the calculations of Ref. 39 using the Drude model. It was noticed²⁴ that the high temperature asymptotics of the Casimir force between real metals computed in Ref. 39 is two times smaller than in the case of an ideal metal — independently of how high the conductivity of the real metal is — which is a nonphysical property. As was noted in Ref. 40, the computations of Ref. 39 are also in contradiction with the experiment.¹⁵

To improve this situation in Ref. 41 a detailed computation of the Casimir force at nonzero temperature were performed based on Drude's model and a Lifshitz formula with some modified zero-frequency term. The modification made is based on a generalization of the prescription of Ref. 22 for the case of real metals. The results of Ref. 41 are in agreement with both limiting cases of an ideal metal and a metal described by the plasma model. In Refs. 42, 43 one more result was obtained for the temperature Casimir force between real metals which, however, disagrees with both the results of Ref. 39 from one side and of Refs. 23, 24 from the other. According to Refs. 42, 43 at small frequencies all real metals are indistinguishable from the ideal metal.

This leads to the absence of any finite conductivity correction to the Casimir force starting from moderate separations of several micrometers between the test bodies. Even more, in the approach of Refs. 42, 43 this property is independent on the quality of the real metal.

As is seen from the above, at the time being there is no agreement in the literature concerning the calculation of the Casimir force acting between real metals at nonzero temperature. Different results are obtained on this subject by different authors starting from one and the same theoretical foundation given by the Lifshitz formula for dielectrics. In the present paper we discuss the scattering formalism of quantum field theory at finite temperature in the Matsubara formulation underlying Lifshitz formula and we argue that it leads to well defined and consistent results both physically and mathematically if the dielectric function is described by the plasma model. This gives additional support to the results of Refs. 23, 24. Contrary, when the dielectric function of the Drude model is used the scattering formalism becomes inconsistent causing the nonphysical results of Refs. 39, 42, 43. In this case the modification of Lifshitz formula suggested in Ref. 41 is needed.

Below, starting from the dielectric function of the plasma model we calculate the Casimir force for the configurations of two parallel plates and a spherical lens (sphere) above a plate made of real metals. The temperature corrections are taken into account completely and the effects of finite conductivity of the boundary metal are treated perturbatively up to second order in some small parameter having the meaning of the relative penetration depth of electromagnetic zero-point oscillations into the metal. Note that in the analytical computations of the previous paper²³ the finite conductivity corrections were calculated up to the first order only, whereas in Ref. 24 both corrections (due to nonzero temperature and finite conductivity) were treated perturbatively. The analytical results obtained below are compared with the results of numerical computations and good agreement is observed for all space separations exceeding the plasma wavelength of the boundary metal.

We also include roughness correction and demonstrate a way how to take into account the influence of real conditions (which include all three types of corrections) onto the Casimir force.

The paper is organized as follows. In Sec.2 the general formalism is presented and the scopes of its consistency are elucidated. In Sec.3 the temperature correction to the Casimir force is calculated for the configuration of two parallel plates up to the second perturbation order in the relative penetration depth. Sec.4 contains the analogous results for the configuration of a sphere (spherical lens) above a plate. In Sec.5 the roughness correction is taken into account along with nonzero temperature and finite conductivity corrections. In Sec.6 the reader finds our conclusions and discussion.

2 General formalism and its scopes

Lifshitz' original derivation³⁸ of the Casimir force at nonzero temperature acting between two dielectric semispaces separated by a gap was based on the assumption that the dielectric materials can be considered as continuous media characterized by randomly fluctuating sources. The modern derivation³⁷ is based on quantum field theory at nonzero temperature, $T \neq 0$, in the Matsubara formulation. Thereby one considers the Euclidean version with the electromagnetic field periodic in the Euclidean time variable within the time interval $\beta = \hbar/(k_B T)$, where k_B and \hbar are the Boltzmann and Planck constants, respectively.

Let us consider two dielectric semispaces with frequency dependent permittivity $\varepsilon(\omega)$ restricted by two planes at $z = \pm a/2$ and separated by a vacuum gap of width a between them. Due to periodicity in the time coordinate, the frequency spectrum is discrete $\omega_l = 2\pi l/\beta$, with $l = \dots -2, -1, 0, 1, 2, \dots$. The calculation of the free energy is reduced to the solution of a one-dimensional scattering problem on the z -axis. In fact an electromagnetic wave which is coming from the left, or from the right, in the dielectric se-

misplaces will be scattered on the vacuum gap and there will be a reflected and a transmitted wave. The free energy of the field per unit area, E_{ss} , is calculated with the help of ζ -regularization method. The result is³⁷

$$E_{ss}(a) = -\frac{\hbar}{2\beta} \sum_l \int \frac{d\mathbf{k}_\perp}{(2\pi)^2} \left[\ln s_{11}^{\parallel}(i\xi_l, \mathbf{k}_\perp) + \ln s_{11}^{\perp}(i\xi_l, \mathbf{k}_\perp) \right], \quad (1)$$

where $s_{11}^{\parallel}(i\xi_l, \mathbf{k}_\perp)$ and $s_{11}^{\perp}(i\xi_l, \mathbf{k}_\perp)$ are the scattering coefficients for parallel and perpendicular polarizations, respectively, $\mathbf{k}_\perp = (k_x, k_y)$ is the wave vector in the planes perpendicular to the z -axis, and $\xi_l = 2\pi l/\beta$. The solution of the scattering problem reads³⁷

$$\begin{aligned} s_{11}^{\parallel} &= \frac{4\varepsilon(i\xi_l)k_l q_l e^{k_l a}}{[\varepsilon(i\xi_l)q_l + k_l]^2 e^{q_l a} - [\varepsilon(i\xi_l)q_l - k_l]^2 e^{-q_l a}}, \\ s_{11}^{\perp} &= \frac{4k_l q_l e^{k_l a}}{(q_l + k_l)^2 e^{q_l a} - (q_l - k_l)^2 e^{-q_l a}}, \end{aligned} \quad (2)$$

where the following notations are introduced

$$q_l = \sqrt{\frac{\xi_l^2}{c^2} + k_\perp^2}, \quad k_l = \sqrt{\varepsilon(i\xi_l) \frac{\xi_l^2}{c^2} + k_\perp^2}, \quad k_\perp \equiv |\mathbf{k}_\perp|. \quad (3)$$

Now we substitute (2) into (1) and perform renormalization in order to get the free energy equal to zero in the case of infinitely far remote plates.²⁸ In terms of reflection coefficients r_{\parallel} , r_{\perp} for the electromagnetic waves of the two different polarizations one obtains

$$\begin{aligned} E_{ss}(a) &= \frac{k_B T}{4\pi} \sum_l \int_0^\infty k_\perp dk_\perp \left\{ \ln [1 - r_{\parallel}^2(\xi_l, k_\perp) e^{-2aq_l}] \right. \\ &\quad \left. + \ln [1 - r_{\perp}^2(\xi_l, k_\perp) e^{-2aq_l}] \right\}, \end{aligned} \quad (4)$$

where

$$r_{\parallel}^2(\xi_l, k_\perp) = \left(\frac{\varepsilon(i\xi_l)q_l - k_l}{\varepsilon(i\xi_l)q_l + k_l} \right)^2, \quad r_{\perp}^2(\xi_l, k_\perp) = \left(\frac{q_l - k_l}{q_l + k_l} \right)^2. \quad (5)$$

The Casimir force per unit area acting between two semispaces is obtained as $-\partial E_{ss}/\partial a$ with the result

$$\begin{aligned} F_{ss}(a) &= -\frac{k_B T}{2\pi} \sum_l \int_0^\infty k_\perp dk_\perp q_l \left\{ \left[r_{\parallel}^{-2}(\xi_l, k_\perp) e^{2aq_l} - 1 \right]^{-1} \right. \\ &\quad \left. + \left[r_{\perp}^{-2}(\xi_l, k_\perp) e^{2aq_l} - 1 \right]^{-1} \right\}. \end{aligned} \quad (6)$$

This equation, up to change of variables, coincides with the original Lifshitz expression³⁸ for the Casimir force between dielectrics at nonzero temperature.

Our aim is to apply Eqs. (4)–(6) in the case of test bodies made of real metals. Then, the zero-frequency contribution for $\xi_0 = 0$ in Eq. (4) may become indefinite in the case of perpendicular polarization when the dielectric permittivity turns into infinity. For example, let us consider the Drude dielectric function on the imaginary axis

$$\varepsilon(i\xi) = 1 + \frac{\omega_p^2}{\xi(\xi + \gamma)}, \quad (7)$$

where ω_p is the plasma frequency, γ is the relaxation frequency, which gives a good approximation of dielectric properties for some metals, e.g., for aluminium. Note that for dielectrics $\varepsilon(i\xi) \rightarrow \varepsilon_0$ when $\xi \rightarrow 0$. It is evident that

$$\xi^2 \varepsilon(i\xi) \rightarrow 0 \quad \text{when } \xi \rightarrow 0, \quad (8)$$

both for Drude's model and dielectrics. By this reason, it follows from Eq. (3) that $q_0 = k_0 = k_\perp$. Strictly speaking, the mathematical derivation leading to Eq. (2) for s_{11}^\perp is inapplicable in that case. Instead, the direct solution of the scattering problem in the case $q_0 = k_0$ gives the result that s_{11}^\perp is arbitrary and $s_{12}^\perp = 0$, where s_{12} is the nondiagonal element of the scattering matrix. In the case of dielectrics the unitarity condition is valid which immediately leads to $|s_{11}^\perp|^2 = 1$ and, due to dispersion relation, to $s_{11}^\perp = 1$. In fact, the same result is obtained from Eq. (2) in the limit $q_0 = k_0$. However, as to the case of the Drude model, which describes a medium with dissipation, the unitarity condition is not applicable, and, therefore, the scattering coefficient s_{11}^\perp remains indefinite. Because of this, the direct application of Lifshitz formula (as in Ref. 39) leads to incorrect results. To apply the Lifshitz formula at nonzero temperature in combination with Drude model some additional prescription is needed to give the definite value to the zero-frequency term (see Ref. 41 for details).

At the same time for the longitudinal polarization the scattering coefficient is well defined in the limit of zero frequency. In the case of the Drude model, up to terms independent of a , it has the limiting value

$$s_{11}^{\parallel} \rightarrow (1 - e^{-2ak_{\perp}})^{-1} \quad \text{when } \xi \rightarrow 0, \quad (9)$$

i.e., the same as for ideal metals. For dielectrics

$$s_{11}^{\parallel} \rightarrow \left[\left(\frac{\varepsilon_0 + 1}{\varepsilon_0 - 1} \right)^2 - e^{-2ak_{\perp}} \right]^{-1} \quad \text{when } \xi \rightarrow 0. \quad (10)$$

Thus, the transition from Eq. (1) to Eqs. (4), (6) is unjustified in the case of Drude's dielectric function. This explains why the nonphysical results were obtained when substituting Eq. (7) into Eq. (6) (see Introduction).

Another model of the dielectric function is the plasma one,

$$\varepsilon(i\xi) = 1 + \frac{\omega_p^2}{\xi^2}, \quad (11)$$

which is the limiting case of (7) when the relaxation frequency γ goes to zero. In the case of the plasma dielectric function

$$\xi^2 \varepsilon(i\xi) \rightarrow \omega_p^2 \neq 0 \quad \text{when } \xi \rightarrow 0. \quad (12)$$

As a consequence, here $q_0 \neq k_0$ and the limiting value of the perpendicular scattering coefficient is given by

$$s_{11}^{\perp} \rightarrow \left[\frac{k_{\perp} + \sqrt{\frac{\omega_p^2}{c^2} + k_{\perp}^2}}{k_{\perp} - \sqrt{\frac{\omega_p^2}{c^2} + k_{\perp}^2}} - e^{-2ak_{\perp}} \right]^{-1} \quad \text{when } \xi \rightarrow 0. \quad (13)$$

The limiting value of the longitudinal scattering coefficient in the case of plasma model is the same as in Eq. (9). Because of this, the scattering problem is well defined for the dielectric function (11) and Eqs. (4) and (6) can be reliably applied to calculate the free energy and the Casimir force. No additional prescriptions or modifications are admissible in the case of plasma model. Because of this, the manipulations of Refs. 42, 43 changing the zero-frequency term of Lifshitz formula for both plasma and Drude models seem

to be unfounded. In the case of the plasma model they lead to disagreement with the results of Refs. 23, 24 where no modifications of Lifshitz formula have been made.

In the next section the analytical computations of the temperature correction to the Casimir force are performed on the basis of Eqs. (4), (6) by using the plasma model (11).

3 Temperature correction to the Casimir force for two parallel plates made of real metal

We start with the Lifshitz formula (6) and rewrite it in terms of dimensionless variables

$$y = 2aq_l = 2a\sqrt{\frac{\xi_l^2}{c^2} + k_\perp^2}, \quad x_l = 2a\frac{\xi_l}{c} \quad (14)$$

resulting in

$$F_{ss}(a) = -\frac{k_B T}{16\pi a^3} \sum_l \int_{|x_l|}^{\infty} y^2 dy \left\{ \left[r_{\parallel}^{-2}(x_l, y) e^y - 1 \right]^{-1} + \left[r_{\perp}^{-2}(x_l, y) e^y - 1 \right]^{-1} \right\}. \quad (15)$$

Here the reflection coefficients in new variables take the form

$$r_{\parallel}(x_l, y) = \frac{\varepsilon y - \sqrt{(\varepsilon - 1)x_l^2 + y^2}}{\varepsilon y + \sqrt{(\varepsilon - 1)x_l^2 + y^2}}, \quad r_{\perp}(x_l, y) = \frac{y - \sqrt{(\varepsilon - 1)x_l^2 + y^2}}{y + \sqrt{(\varepsilon - 1)x_l^2 + y^2}}, \quad (16)$$

and $\varepsilon \equiv \varepsilon[icx_l/(2a)]$.

To separate within (15) the contribution of temperature $T = 0$ and the temperature correction one can use the representation of this formula in terms of continuous x instead of discrete summation in x_l . Applying the Poisson summation formula one obtains from (15)^{20,22,23}

$$F_{ss}(a) = -\frac{\hbar c}{32\pi^2 a^4} \sum_l \int_0^{\infty} y^2 dy \int_0^y dx \cos(ltx) \times \left\{ \left[r_{\parallel}^{-2}(x, y) e^y - 1 \right]^{-1} + \left[r_{\perp}^{-2}(x, y) e^y - 1 \right]^{-1} \right\}, \quad (17)$$

where $t = T_{eff}/T$, $k_B T_{eff} \equiv \hbar c/(2a)$. The reflection coefficients preserve their form (16) with a change $x_l \rightarrow x$.

In Eq. (17) the term with $l = 0$ is the Casimir force at zero temperature,²² whereas the terms with $l \neq 0$ represent the temperature corrections. For further needs it is convenient to write the temperature correction of the Casimir force acting between real metals as a sum of longitudinal and perpendicular contributions,

$$\Delta_T F_{ss}(a) = \Delta_T^{\parallel} F_{ss}(a) + \Delta_T^{\perp} F_{ss}(a), \quad (18)$$

where

$$\begin{aligned} \Delta_T^{\parallel(\perp)} F_{ss}(a) = & -\frac{\hbar c}{32\pi^2 a^4} \sum_{l=1}^{\infty} \int_0^{\infty} y^2 dy \int_0^y dx \cos(ltx) \\ & \times \left[r_{\parallel(\perp)}^{-2}(x, y) e^y - 1 \right]^{-1}. \end{aligned} \quad (19)$$

To compute the temperature correction (18), (19) one should use some specific functional dependence of the dielectric permittivity on the frequency. Here we use the plasma model (11) for which the theory is well defined (see Sec.2). In terms of dimensionless variables the dielectric function (11) is given by

$$\varepsilon = \varepsilon(x) = 1 + \frac{\tilde{\omega}_p^2}{x^2} = 1 + \frac{4a^2}{\delta_0^2 x^2}, \quad (20)$$

where $\tilde{\omega}_p = 2a\omega_p/c$, $\delta_0 = c/\omega_p$ is the effective penetration depth of electromagnetic zero-point oscillations into the real metal. For the space separations between the plates $a \gg \delta_0$ the natural small parameter is $\delta_0/a \ll 1$. In fact this condition is valid for $a > \lambda_p$, where $\lambda_p = 2\pi c/\omega_p$ is the effective plasma wavelength, since $\delta_0 = \lambda_p/(2\pi)$.

Here we calculate the temperature correction (18), (19) analytically taking completely into account the nonzero temperature and using the perturbation theory up to the second power in the small parameter δ_0/a in order to take approximate account of the finite conductivity of the boundary metal. To perform the computations let us expand first the expressions in Eq. (19),

containing reflection coefficients, in powers of δ_0/a . The result is

$$\begin{aligned} \left[r_{\parallel}^{-2}(x, y) e^y - 1 \right]^{-1} &= \frac{1}{e^y - 1} - 2 \frac{\delta_0}{a} \frac{x^2 e^y}{y(e^y - 1)^2} \\ &\quad + 2 \left(\frac{\delta_0}{a} \right)^2 \frac{x^4 e^y (e^y + 1)}{y^2 (e^y - 1)^3} + O \left(\frac{\delta_0^3}{a^3} \right), \\ \left[r_{\perp}^{-2}(x, y) e^y - 1 \right]^{-1} &= \frac{1}{e^y - 1} - 2 \frac{\delta_0}{a} \frac{y e^y}{(e^y - 1)^2} \\ &\quad + 2 \left(\frac{\delta_0}{a} \right)^2 \frac{y^2 e^y (e^y + 1)}{(e^y - 1)^3} + O \left(\frac{\delta_0^3}{a^3} \right). \end{aligned} \quad (21)$$

Substituting (21) into (19) and calculating integrals with the help of the formulas 3.951(12, 13) of Ref. 44 one obtains finally the contribution of the longitudinal mode

$$\begin{aligned} \Delta_T^{\parallel} F_{ss}(a) &= -\frac{\hbar c}{16\pi^2 a^4} \sum_{l=1}^{\infty} \left\{ \frac{1}{(lt)^4} - \frac{\pi^3 \coth(\pi lt)}{lt \sinh^2(\pi lt)} \right. \\ &\quad + 2 \frac{\delta_0}{a} \left[\frac{\pi}{(lt)^3} \coth(\pi lt) + \frac{\pi^2}{(lt)^2 \sinh^2(\pi lt)} (1 + \pi lt \coth(\pi lt) \right. \\ &\quad \left. \left. + (\pi lt)^2 - 3(\pi lt)^2 - \coth^2(\pi lt) \right) \right] \\ &\quad + 2 \left(\frac{\delta_0}{a} \right)^2 \frac{\pi^3}{\sinh^2(\pi lt)} [-4\pi + 12\pi \coth^2(\pi lt) + 7\pi^2 lt \coth(\pi lt) \\ &\quad \left. \left. - 12\pi^2 lt \coth^3(\pi lt) \right] \right\}. \end{aligned} \quad (22)$$

Quite analogously the contribution of the perpendicular mode is

$$\begin{aligned} \Delta_T^{\perp} F_{ss}(a) &= -\frac{\hbar c}{16\pi^2 a^4} \sum_{l=1}^{\infty} \left\{ \frac{1}{(lt)^4} - \frac{\pi^3 \coth(\pi lt)}{lt \sinh^2(\pi lt)} \right. \\ &\quad + 2 \frac{\delta_0}{a} \frac{\pi^3}{lt \sinh^2(\pi lt)} [3 \coth(\pi lt) + \pi lt - 3\pi lt \coth^2(\pi lt)] \\ &\quad + 8 \left(\frac{\delta_0}{a} \right)^2 \frac{\pi^3}{lt \sinh^2(\pi lt)} [-3 \coth(\pi lt) + 2(\pi lt)^2 \coth^2(\pi lt) \\ &\quad \left. \left. - 2\pi lt + 6\pi lt \coth^2(\pi lt) - 3(\pi lt)^2 \coth^3(\pi lt) \right] \right\}. \end{aligned} \quad (23)$$

Finally, the total temperature correction (18) is given by

$$\Delta_T F_{ss}(a) = -\frac{\hbar c}{8\pi^2 a^4} \sum_{l=1}^{\infty} \left\{ \frac{1}{(lt)^4} - \frac{\pi^3 \coth(\pi lt)}{lt \sinh^2(\pi lt)} \right.$$

$$\begin{aligned}
& + \frac{\delta_0}{a} \frac{\pi^3}{lt \sinh^2(\pi lt)} \left[\frac{1}{(\pi lt)^2} \sinh(\pi lt) \cosh(\pi lt) + 4 \coth(\pi lt) \right. \\
& \quad \left. + 2\pi lt - 6\pi lt \coth^2(\pi lt) + \frac{1}{\pi lt} \right] \\
& + 3 \left(\frac{\delta_0}{a} \right)^2 \frac{\pi^3}{lt \sinh^2(\pi lt)} \left[-4\pi lt + 5(\pi lt)^2 \coth(\pi lt) + 12\pi lt \coth^2(\pi lt) \right. \\
& \quad \left. - 8(\pi lt)^2 \coth^3(\pi lt) - 4 \coth(\pi lt) \right] \Bigg\}.
\end{aligned} \tag{24}$$

Note that the first two terms on the right-hand side of (24) which are of zeroth order in δ_0/a coincide with the well-known result for the ideal metal^{21,22}, whereas the coefficient of the first power in δ_0/a was first obtained in Ref. 23.

In the case of low temperatures (small separations), $t \gg 1$, one can substitute the hyperbolic functions by their asymptotics. Preserving the largest of the exponentially small contributions and performing summations of the power ones we obtain from (24)

$$\begin{aligned}
\Delta_T F_{ss}(a) \approx & -\frac{\hbar c}{8\pi^2 a^4} \left\{ \frac{\pi^4}{90t^4} - \frac{4\pi^3}{t} e^{-2\pi t} \right. \\
& \left. + \frac{\delta_0}{a} \left[\frac{\pi}{t^3} \zeta(3) - 16\pi^4 e^{-2\pi t} \right] - 36\pi^5 t \left(\frac{\delta_0}{a} \right)^2 e^{-2\pi t} \right\},
\end{aligned} \tag{25}$$

where $\zeta(z)$ is Riemann's zeta function. It is seen that the second perturbative order in δ_0/a is exponentially small in t and does not contain purely powers in t contributions like the first order term of (25). This is in agreement with the perturbation results of Ref. 24 where the double perturbation theory in the small parameters δ_0/a and $1/t$ was developed. At the same time, under the natural supposition $(\delta_0/a)t \sim 1$ the second order term turns out to be approximately 90 times larger than the exponentially small contribution in the zeroth order term and 7 times larger than the exponentially small contribution in the first order term.

Now consider the case of high temperatures (large separations) when $t \ll 1$. It is more simple to extract it not from Eq. (24) but directly from Eqs. (18), (19), (21). To do this one should perform the integration with respect to x

in the same way as above and then change the order of summation and integrations with respect to y . Due to the smallness of t all summations can be performed by the use of the formula⁴⁴

$$\sum_{l=0}^{\infty} \frac{\sin(lty)}{l} = \frac{\pi - ty}{2}, \quad (26)$$

which is valid for $0 < ty < 2\pi$. In the further integrations with respect to y all functions under the integrals decrease with y as $\exp(-y)$, so that the infinite upper limit of the integration can be changed for $\tilde{y} = (2\pi/t) - \alpha$, where $\alpha > 0$, with the required accuracy. As a result, the high temperature limit of the temperature correction to the Casimir force between real metals is given by

$$\begin{aligned} \Delta_T F_{ss}(a) \approx & -\frac{\hbar c}{8\pi^2 a^4} \left\{ \frac{\pi \zeta(3)}{t} - \frac{\pi^4}{30} \right. \\ & \left. + \frac{\delta_0}{a} \left[-\frac{3\pi}{t} \zeta(3) + \frac{8\pi^4}{45} \right] + \left(\frac{\delta_0}{a} \right)^2 \left[\frac{12\pi}{t} \zeta(3) - \frac{4\pi^4}{5} \right] \right\}. \end{aligned} \quad (27)$$

Let us discuss the application range of the analytical result (24) and the asymptotic representations (25) and (27). Bearing in mind that according to Eq. (17) the Casimir force at nonzero temperature and finite conductivity is given by

$$F_{ss}(a) = F_{ss}(a; T = 0) + \Delta_T F_{ss}(a), \quad (28)$$

it is convenient to compute the quantity

$$k_{ss} = \frac{\Delta_T F_{ss}(a)}{F_{ss}(a; T = 0)}. \quad (29)$$

Then the value of $(1 + k_{ss})$ has the meaning of a correction factor. Indeed, multiplying the Casimir force $F_{ss}(a; T = 0)$ computed with account of finite conductivity at zero temperature by $(1 + k_{ss})$ one obtains the Casimir force at both nonzero temperature and finite conductivity. Note that $F_{ss}(a; T = 0)$ was computed in a number of papers^{27–29}, and below we use the numerical and analytical results obtained there to calculate k_{ss} .

In Table 1 the values of k_{ss} are presented at several separations computed by the use of Eq. (24) (second column), by the use of low-temperature asymptotics (third column), and high-temperature asymptotics (fourth column). In all computations, as an example, the value of the plasma frequency $\omega_p = 12.5 \text{ eV}$ is used as for aluminium⁴⁵ (this corresponds to the plasma wavelength of approximately $\lambda_p \approx 99 \text{ nm}$). It is seen from Table 1 that the asymptotic of low temperatures gives the same values of k_{ss} as Eq. (24) at all separations $a \leq 2 \mu\text{m}$ (for $a > 3 \mu\text{m}$ it is not applicable). Comparing data of columns two and four one can conclude that the asymptotics of high temperatures works good for $a \geq 7 \mu\text{m}$ and is not applicable for $a < 5 \mu\text{m}$. In the transition region $3 \mu\text{m} \leq a \leq 5 \mu\text{m}$ neither of the asymptotics but Eq. (24) itself should be used to compute the temperature correction to the Casimir force acting between real metals. It is noticeable, that data of column 2 are practically the same irrespective of whether one or two perturbation orders are taken into account. These data coincide also with the results of numerical computations by Eqs. (18)–(20) in all separation range $0.1 \mu\text{m} \leq a \leq 10 \mu\text{m}$.

It is interesting also to discuss the comparative contribution to the temperature correction which results from the longitudinal and perpendicular modes given by Eqs. (22), (23). At small separations the contribution of $\Delta_T^{\parallel} F_{ss}$ to the temperature correction dominates the contribution of $\Delta_T^{\perp} F_{ss}$. By way of example, at $a = 0.1 \mu\text{m}$ one has $\Delta_T^{\parallel} F_{ss} / \Delta_T^{\perp} F_{ss} = 43.1$. This ratio, however, quickly decreases with the increase of space separation. Thus, at $a = 0.3 \mu\text{m}$ it is equal to 5.67, whereas at $a = 0.5 \mu\text{m}$ and $a = 0.7 \mu\text{m}$ it equals to 2.68 and 1.86, respectively. At large separations both modes give almost equivalent contribution to the temperature correction. For example, at $a = 7 \mu\text{m}$ the abovementioned ratio is equal to 1.015 and at $a = 10 \mu\text{m}$ it is equal to 1.01.

4 Temperature correction for a sphere above a plate made of real metal

In most of the experiments on measurement of the Casimir force the configuration of a sphere (spherical lens) placed above a plate (semispace) is used^{15–18} (in fact the configuration of two crossed cylinders, as in Ref. 19, is equivalent to it). The expression for the Casimir force at nonzero temperature acting in this configuration can be obtained by means of the proximity force theorem⁴⁶

$$F_{sl}(a) = 2\pi R E_{ss}(a), \quad (30)$$

where $E_{ss}(a)$ is the free energy per unit area of the two plates defined in Eq. (4), R is the curvature radius of the sphere (spherical lens). In terms of the dimensionless variables of Eq. (14) the force acting between a lens and a plate is

$$F_{sl}(a) = \frac{k_B T R}{8a^2} \sum_l \int_{|x_l|}^{\infty} y dy \left\{ \ln [1 - r_{||}^2(x_l, y) e^{-y}] + \ln [1 - r_{\perp}^2(x_l, y) e^{-y}] \right\}, \quad (31)$$

where the reflection coefficients $r_{||(\perp)}$ are defined in (16).

After applying the Poisson summation formula, Eq. (31) can be rewritten in the form analogical to Eq. (17)

$$F_{sl}(a) = \frac{\hbar c R}{16\pi a^3} \sum_l \int_0^{\infty} y dy \int_0^y dx \cos(ltx) \times \left\{ \ln [1 - r_{||}^2(x, y) e^{-y}] + \ln [1 - r_{\perp}^2(x, y) e^{-y}] \right\}. \quad (32)$$

Once more, the term with $l = 0$ is the Casimir force at zero temperature, the terms with $l \neq 0$ represent the temperature corrections to it. In accordance with Eq. (18) the temperature correction can be splitted into a sum of longitudinal and perpendicular contributions (with a change of index $ss \rightarrow sl$) expressed by

$$\Delta_T^{||(\perp)} F_{sl}(a) = \frac{\hbar c R}{8\pi a^3} \sum_{l=1}^{\infty} \int_0^{\infty} y dy \int_0^y dx \cos(ltx)$$

$$\times \ln [1 - r_{||(\perp)}^2 (x, y) e^{-y}]. \quad (33)$$

To compute the temperature correction analytically by using the plasma model we substitute Eq. (20) into Eq. (33) and, again, expand into powers of the small parameter δ_0/a

$$\begin{aligned} \ln [1 - r_{||}^2 (x, y) e^{-y}] &= \ln(1 - e^{-y}) + 2 \frac{\delta_0}{a} \frac{x^2}{y(e^y - 1)} \\ &\quad - 2 \left(\frac{\delta_0}{a} \right)^2 \frac{x^4 e^y}{y^2 (e^y - 1)^2} + O \left(\frac{\delta_0^3}{a^3} \right), \\ \ln [1 - r_{\perp}^2 (x, y) e^{-y}] &= \ln(1 - e^{-y}) + 2 \frac{\delta_0}{a} \frac{y}{e^y - 1} \\ &\quad - 2 \left(\frac{\delta_0}{a} \right)^2 \frac{y^2 e^y}{(e^y - 1)^2} + O \left(\frac{\delta_0^3}{a^3} \right). \end{aligned} \quad (34)$$

Let us calculate first the longitudinal temperature correction which is obtained by the substitution of the first equality from (34) into (33). All integrals with respect to x are trivial. The resulting integrals with respect to y can be found in Ref. 44 (formulas 3.951(12, 13)) except of the following one which, however, also can be computed analytically:

$$\begin{aligned} &\int_0^\infty \frac{dy}{y} \frac{e^y}{(e^y - 1)^2} [\sin(lty) - lty \cos(lty)] \\ &= \frac{\pi l^2 t^2}{4} \coth(\pi lt) - \frac{1}{4\pi} \left[\frac{\pi^2}{6} + 2\pi lt \ln(1 - e^{-2\pi lt}) - \frac{2\pi^2 l^2 t^2}{e^{2\pi lt} - 1} - \text{Li}_2(e^{-2\pi lt}) \right], \end{aligned} \quad (35)$$

where $\text{Li}_2(z)$ is the polylogarithm function. As a result the contribution of the longitudinal modes to the temperature correction is

$$\begin{aligned} \Delta_{TFsl}^{\parallel}(a) &= -\frac{\hbar c R}{8\pi a^3} \sum_{l=1}^{\infty} \left\{ \frac{\pi}{2(lt)^3} \coth(\pi lt) - \frac{1}{(lt)^4} + \frac{\pi^2}{2(lt)^2} \frac{1}{\sinh^2(\pi lt)} \right. \\ &\quad + 2 \frac{\delta_0}{a} \left[\frac{\pi}{(lt)^3} \coth(\pi lt) - \frac{3}{(lt)^4} + \frac{\pi^2}{(lt)^2} \frac{1}{\sinh^2(\pi lt)} + \frac{\pi^3 \coth(\pi lt)}{lt \sinh^2(\pi lt)} \right] \\ &\quad - 2 \left(\frac{\delta_0}{a} \right)^2 \left[\frac{\pi}{(lt)^5} + \frac{\pi^4}{\sinh^2(\pi lt)} \left(1 - 3 \coth^2(\pi lt) - \frac{\coth(\pi lt)}{\pi lt} - \frac{2}{(\pi lt)^2} \right) \right. \\ &\quad \left. \left. + \frac{6}{\pi(lt)^5} \left(2\pi lt \ln(1 - e^{-2\pi lt}) - \frac{2\pi^2 (lt)^2}{e^{2\pi lt} - 1} - \text{Li}_2(e^{-2\pi lt}) \right) \right] \right\}. \end{aligned} \quad (36)$$

The contribution of the perpendicular modes is calculated simply as

$$\begin{aligned} \Delta_T^\perp F_{sl}(a) = & -\frac{\hbar c R}{8\pi a^3} \sum_{l=1}^{\infty} \left\{ \frac{\pi}{2(lt)^3} \coth(\pi lt) - \frac{1}{(lt)^4} + \frac{\pi^2}{2(lt)^2} \frac{1}{\sinh^2(\pi lt)} \right. \\ & + 2\frac{\delta_0}{a} \left[\frac{\pi^3}{lt} \frac{\coth(\pi lt)}{\sinh^2(\pi lt)} - \frac{1}{(lt)^4} \right] \\ & \left. - 2\left(\frac{\delta_0}{a}\right)^2 \frac{\pi^3}{lt \sinh^2(\pi lt)} [3 \coth(\pi lt) + \pi lt - 3\pi lt \coth^2(\pi lt)] \right\}. \end{aligned} \quad (37)$$

Putting together the contributions of both modes the total temperature correction for the configuration of a lens above a plate is obtained:

$$\begin{aligned} \Delta_T F_{sl}(a) = & -\frac{\hbar c R}{4\pi a^3} \sum_{l=1}^{\infty} \left\{ \frac{\pi}{2(lt)^3} \coth(\pi lt) - \frac{1}{(lt)^4} + \frac{\pi^2}{2(lt)^2} \frac{1}{\sinh^2(\pi lt)} \right. \\ & + \frac{\delta_0}{a} \left[\frac{\pi}{(lt)^3} \coth(\pi lt) - \frac{4}{(lt)^4} + \frac{\pi^2}{(lt)^2} \frac{1}{\sinh^2(\pi lt)} + \frac{2\pi^3}{lt} \frac{\coth(\pi lt)}{\sinh^2(\pi lt)} \right] \\ & - \left(\frac{\delta_0}{a}\right)^2 \left[\frac{\pi}{(lt)^5} + \frac{2\pi^4}{\sinh^2(\pi lt)} \left(1 - 3 \coth^2(\pi lt) + \frac{\coth(\pi lt)}{\pi lt} - \frac{1}{(\pi lt)^2} \right) \right. \\ & \left. \left. + \frac{6}{\pi(lt)^5} \left(2\pi lt \ln(1 - e^{-2\pi lt}) - \frac{2\pi^2(lt)^2}{e^{2\pi lt} - 1} - \text{Li}_2(e^{-2\pi lt}) \right) \right] \right\}. \end{aligned} \quad (38)$$

The terms of zeroth order in δ_0/a in the right-hand side of (38) coincide with the known result for an ideal metal.³⁷ The coefficient of the first power in δ_0/a was already obtained in Ref. 23.

Now, let us consider the limiting cases of Eq. (38) corresponding to low and high temperatures (small and large separations). At low temperatures, $t \gg 1$, and, preserving the largest of the exponentially small contributions, one obtains from Eq. (38)

$$\begin{aligned} \Delta_T F_{sl}(a) \approx & -\frac{\hbar c R}{4\pi a^3} \left\{ \frac{\pi \zeta(3)}{2t^3} - \frac{\pi^4}{90t^4} + \frac{2\pi^2}{t^2} e^{-2\pi t} \right. \\ & \left. + \frac{\delta_0}{a} \left[\frac{\pi}{t^3} \zeta(3) - \frac{2\pi^4}{45t^4} + \frac{8\pi^3}{t} e^{-2\pi t} \right] - \left(\frac{\delta_0}{a}\right)^2 \left[\frac{\pi \zeta(5)}{t^5} - 16\pi^4 e^{-2\pi t} \right] \right\}. \end{aligned} \quad (39)$$

It is noticeable, that for the configuration of a lens above a plate the second perturbative order in δ_0/a contains power-type contributions in t , not only exponentially small ones. It is, however, of order t^{-5} in agreement with

Ref. 24 where the absence of temperature corrections of powers lower than $1/t^5$ was proved for the perturbation orders $(\delta_0/a)^k$ with $k = 2, 3, 4, 5, 6$.

The limit of high temperatures can be obtained in the same way as in Sec.3, i.e., starting from Eq. (33) and using Eq. (26). The result is

$$\Delta_T F_{sl}(a) \approx -\frac{\hbar c R}{4\pi a^3} \left\{ \frac{\pi \zeta(3)}{2t} - \frac{\pi^4}{90} + \frac{\delta_0}{a} \left[-\frac{\pi \zeta(3)}{t} + \frac{2\pi^4}{45} \right] + \left(\frac{\delta_0}{a} \right)^2 \left[\frac{3\pi \zeta(3)}{t} - \frac{4\pi^4}{25} \right] \right\}. \quad (40)$$

To determine the range of applicability of both asymptotic representations we compute the quantity

$$k_{sl} = \frac{\Delta_T F_{sl}(a)}{F_{sl}(a; T = 0)}, \quad (41)$$

where $F_{sl}(a; T = 0)$ is the force at zero temperature calculated with account of finite conductivity.^{27–29} The value of $(1 + k_{sl})$ has the meaning of a correction factor to it. The total Casimir force acting between real metals at nonzero temperature is given by

$$F_{sl}(a) = (1 + k_{sl}) F_{sl}(a; T = 0). \quad (42)$$

In Table 2 the values of k_{sl} are presented for aluminium computed (i) by Eq. (38) (second column), (ii) by the low-temperature asymptotic (third column), and (iii) by the high-temperature asymptotic (fourth column). In analogy with the case of two parallel plates, the asymptotics of low and high temperatures work good at separations $a \leq 2 \mu\text{m}$ and $a \geq 6 \mu\text{m}$, respectively. The data of column 2 are practically the same in first- and second-order perturbation theory. They coincide also with the results of numerical computations by the use of Eq. (33).

The comparative contribution of the longitudinal and perpendicular modes to the temperature correction to the Casimir force (see Eqs. (36), (37)) is different in comparison with the case of two plates. Here, $\Delta_T^{\parallel} F_{sl} / \Delta_T^{\perp} F_{sl} = 1.64$ at $a = 0.1 \mu\text{m}$, 1.21 at $a = 0.3 \mu\text{m}$, and 1.13 at $a = 0.5 \mu\text{m}$. This ratio

decreases slowly to the value 1.006 at $a = 10 \mu\text{m}$. Thus, the contributions of both modes are approximately equal to each other at all separations (for two plates the contribution of the longitudinal mode significantly dominates at smallest separations). This is explained by the presence of the term $\sim t^{-3}$ in the zeroth order contribution to $\Delta_T F_{sl}$ (in the case of two plates the zeroth order contribution is $\sim t^{-4}$).

5 Casimir force with account of roughness

The above Eqs. (28), (41) give us the Casimir force computed at nonzero temperature with account of finite conductivity of the boundary metal. Except for the finite value of conductivity, real metallic boundaries are characterized also by some surface roughness. According to the results of Ref. 35 obtained at zero temperature, for a wide range of surface roughness it can be taken into account by some kind of geometrical approach using the averaging of the Casimir force over the rough surface. Here, we generalize this approach for the case of nonzero temperature.

Let two large metallic plates of dimension $L \times L$ be covered by small roughness. Then the distance between two points of the boundary surfaces of different plates with the coordinates (x, y) can be expressed as

$$a(x, y) = a_0 + f(x, y). \quad (43)$$

Here, $f(x, y)$ is simply expressed by the functions describing roughness on both surfaces. The mean distance between the plates a_0 is defined in such a way that

$$\int_{-L}^L dx \int_{-L}^L dy f(x, y) = 0. \quad (44)$$

As a result, the Casimir force taking all real conditions into account (i.e. nonzero temperature, finite conductivity of a metal and surface roughness)

is given as

$$F_{ss}^r(a_0) = \frac{1}{L^2} \int_{-L}^L dx \int_{-L}^L dy F_{ss}[a(x, y)]. \quad (45)$$

Remind that F_{ss} here is given by Eq. (28) with a change of separation distance a for the one defined in Eq. (43). The analogical result can be obtained for the configuration of a sphere above a plate by the use of the proximity force theorem.⁴⁶ As is shown in Ref. 37, Eq. (45) gives the same results as the more fundamental methods for accounting roughness, e.g., based on the specific forms of interatomic potentials or Green's function method.

Finite conductivity and temperature corrections to the Casimir force were computed in Secs.3,4 in the whole distance range $0.1 \mu\text{m} \leq a \leq 10 \mu\text{m}$. Surface roughness makes the most important contribution to the Casimir force for separation distances $a \leq 1 \mu\text{m}$. For such distances (and also for $1 \mu\text{m} < a < 3 \mu\text{m}$) the asymptotic of low temperatures of Eq. (25) can be substituted into Eq. (45) to calculate the Casimir force under real conditions. As to the transition region $3 \mu\text{m} \leq a \leq 5 \mu\text{m}$, the more exact Eq. (24) should be used there (at larger separations roughness corrections are negligible).

6 Conclusions and discussion

As was argued above, the dielectric function as it results from the plasma model can be reliably used to calculate the Casimir force acting between real metals at nonzero temperature. The scattering theory underlying the Lifshitz formula is well defined in the case of the plasma model and its application is straightforward. No additional prescriptions are needed like those formulated in Ref. 22 for ideal metal or in Ref. 41 for real metals described by the Drude dielectric function.

We calculated the temperature correction to the Casimir force between real metals in the configuration of two parallel plates (two semispaces) and for a lens (sphere) above a plate. The analytical expressions for these cor-

rections were obtained which are exact with respect to the temperature but perturbative with respect to the effects of finite conductivity. These effects were taken into account up to the second order in a small parameter having the meaning of the relative penetration depth of electromagnetic zero-point oscillations into the metal. The asymptotics of the obtained expressions were presented at both low and high temperatures relative to $k_B T_{eff} = \hbar c / (2a)$. The asymptotical formulas are in good agreement with the exact ones except of a narrow transition region between the cases of low and high temperatures. The scopes of these regions are determined. In a wide separation range from $0.1 \mu\text{m}$ till $10 \mu\text{m}$ the obtained analytical results are in perfect accordance with the results of numerical computations performed earlier. The comparative contributions of the longitudinal and perpendicular modes to the temperature correction were determined. Modification of the obtained results taking the surface roughness into account was given. This together permits to evaluate the Casimir force under the influence of real conditions which include nonzero temperature, finite conductivity of the boundary metal and surface roughness with a precision of several percent.

The above results are topical ones for the precision measurements of the Casimir force. In view of fundamental and technological applications of the Casimir effect mentioned in Introduction, there is a great theoretical challenge to account for real experimental conditions. In fact, a theory is required which makes it possible to calculate the Casimir force with a precision of being better than one percent. For this purpose one should especially examine the optical properties of the test bodies in use, their surface roughness and take into account the spatial dispersion (in the case when there are thin layers covering the test bodies). The more precise future theory should take into account also the effects of dissipation which are neglected in the presently used Lifshitz formula.

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Table 1: The values of k_{ss} computed by the use of Eq. (24) in comparison with the asymptotic values at low [Eq. (25)] and high [Eq. (27)] temperatures.

| a (μm) | k_{ss} computed by Eq. (24) | k_{ss} at low temperatures | k_{ss} at high temperatures |
|--------------------------|----------------------------------|---------------------------------|----------------------------------|
| 0.1 | 6.58×10^{-6} | 6.58×10^{-6} | |
| 0.3 | 5.48×10^{-5} | 5.48×10^{-5} | |
| 0.5 | 2.12×10^{-4} | 2.12×10^{-4} | |
| 0.7 | 6.05×10^{-4} | 6.05×10^{-4} | |
| 0.9 | 1.42×10^{-3} | 1.42×10^{-3} | |
| 2.0 | 2.74×10^{-2} | 2.76×10^{-2} | |
| 3.0 | 0.1228 | 0.1339 | |
| 4.0 | 0.3100 | 0.4159 | 0.2302 |
| 5.0 | 0.5630 | | 0.5349 |
| 6.0 | 0.8487 | | 0.8400 |
| 7.0 | 1.147 | | 1.144 |
| 8.0 | 1.450 | | 1.449 |
| 10 | 2.059 | | 2.059 |

Table 2: The values of k_{sl} computed by the use of Eq. (38) in comparison with the asymptotic values at low [Eq. (39)] and high [Eq. (40)] temperatures.

| a (μm) | k_{sl} computed by Eq. (38) | k_{sl} at low temperatures | k_{sl} at high temperatures |
|--------------------------|----------------------------------|---------------------------------|----------------------------------|
| 0.1 | 6.69×10^{-5} | 6.70×10^{-5} | |
| 0.3 | 1.08×10^{-3} | 1.08×10^{-3} | |
| 0.5 | 4.33×10^{-3} | 4.33×10^{-3} | |
| 0.7 | 1.09×10^{-2} | 1.09×10^{-2} | |
| 0.9 | 2.181×10^{-2} | 2.181×10^{-2} | |
| 2.0 | 0.1829 | 0.1828 | |
| 3.0 | 0.4810 | 0.4762 | 0.3857 |
| 4.0 | 0.8736 | 0.8113 | 0.8428 |
| 5.0 | 1.309 | | 1.300 |
| 6.0 | 1.759 | | 1.757 |
| 7.0 | 2.215 | | 2.214 |
| 8.0 | 2.671 | | 2.671 |
| 10 | 3.585 | | 3.585 |